Estimating Default Probabilities Implicit in Commercial Mortgage Backed Securities (CMBS)*

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Abstract This paper uses a structural credit risk model, providing an analytical formula to estimate default probabilities implicit in commercial mortgage backed security prices. Empirical studies on CMBS default have focused on the probability of default depending on loan characteristics at the origination and market indices. Recent studies show that unobservable current loan-to-value (LTV) ratio is a key state variable driving default. We update this variable using Real Estate Investment Trust (REIT) property-type indices over time. Later, we employ first passage time approach to study CMBS default using implied LTV.

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Introduction

> *"The subprime crisis has not been averted. In fact, it is still largely ahead of us. The downgrades represent only a small fraction — about 2 percent of the mortgage-backed securities rated for the year between the fourth quarters of 2005 and 2006 — of what the rating agencies suggest could be a mountain of bad debt held by investors, including pension plans, banks and insurance companies. The agencies are primarily downgrading assets with expected losses that are already working their way through the pipeline. They are not projecting future losses."* *The New York Times, July 25 2007.*

Recent subprime crises put rating agencies under scrutiny for calculating subordination levels, driven from default probabilities, for mortgage-backed securities. If this is true for subprime mortgages, it also cast some doubt about commercial mortgage backed securities. In this paper, we estimate the default probabilities implicit in commercial mortgage backed securities using a simple structural model.

Structural and reduced form models are the two primary model describing default process in the literature. While the structural model defines default time using the firm value process, the reduced form approach determines default time by the first jump time of a point process. In structural approach, if the total value of the firm's asset is less than the face value of its debt, firm defaults and debt holders receive the total value of the firm at maturity. The payoff to the liability holders can be viewed as the face value of the loan less a put option with strike equal to the face value of the debt. This is the reason why structural model is also referred as the option theoretical approach. This approach first introduced by

Merton (1974), and default is only allowed at the debt maturity. The most idealized forms of structural default models predict termination to be the rational decision of the borrower in response to evolving financial circumstances, typically represented by stochastic processes describing the value of the underlying asset, for example the building itself (see Kau and Keenan (1995) for a survey of this literature). A second approach, within the structural framework, extends the original Merton model by allowing the default to occur not only at the debt's maturity, but also prior to this date. This approach, also called first passage time model, was first introduced by Black and Cox (1976) and later extended depending on default being exogenous or endogenously determined.

Second approach for modeling default was introduced by Jarrow and Turnbull (1995) and Duffie and Singleton (1999) in finance literature. Default time is defined as the first jump time of a point process. Reduced form models, which do not seek to explain default at all, being content, rather, to incorporate data in order to predict the likelihood of mortgage termination, according to the statistical pattern by which such default apparently occurs. These empirical efforts commonly take the form of duration models, the proportional hazard framework being particularly popular. Archer *et al.* (2002), Ciochetti *et al.* (2002), Ciochetti *et al.* (2003), Deng *et al.* (2005), Vandell (1992), and Yildirim (2007) are some of the many papers used duration models in estimating default probabilities. Such studies find that loan-to-value is positively correlated with default probabilities, and that current loan-to-value is especially important, dominating other variables in terms of its significance in default prediction. The baseline probability of default at any time is then simply shifted up or down by the values of various covariates evaluated at that time. This, though, makes such models of limited usefulness for predicting the future probability of default, starting from, say, the origination of the loan, when those future covariate values are yet unknown. More quantitative version of reduced form models using the cox processes were introduced the real estate literature by Kau *et al.* (2006) and Christopoulos *et al.* (2007).

We employ the first passage time approach in estimating default probabilities implicit in CMBS using implied LTV under the Kau *et al.* (1999) framework. The procedure employed is easily described: a mortgage is idealized to be a consol having a perpetual put option representing the opportunity to default. This thereby yields a simple closed-form formula for the optimal default barrier, one that does not change over time within a stationary environment. The one unknown parameter determining the barrier is then inferred by matching the model to actual default experience, in our case making use of an available body of commercial loans observed between 1998 and 2005, coming from data on commercial backed mortgages (CMBS). The key state variable driving default, the current loan-to-value ratio *(LTV)* of the loan, is updated using Real Estate Investment Trust (REIT) property-type indices over time. The same data source is also used to infer the parameters driving the *LTV* processes by property type and region. Given a loan's calibrated barrier and its associated *LTV* process, it is then a straightforward exercise to obtain the loan's likelihood of future default. The resulting estimations of loan default are thus sensitive to the property type, the region, the contract rate (i.e. cap rate), the interest rate at origination, and the time to maturity for any commercial mortgage within the class being considered.

An outline for this paper is as follows. Section 2 presents the structural model and derives the testable implication. Section 3 provides a description of the data and estimates the model. A conclusion is provided in Section 4.

Modelling Default Probabilities

Once we have identified LTV as the key variable driving default, it is natural that the act of default be triggered by LTV's first passage above a critical threshold, or barrier. Let τ represent the time of default, defined as the first time the underlying process, LTV, crosses the barrier b; that is

$$\tau = \inf\{t \geq 0 : LTV_t \geq b\}, \tag{1}$$

where $b > 0$ (See Figure 1.). Let LTV follows geometric Brownian motion of the form,

$$\frac{dLTV_t}{LTV_t} = \mu_\ell dt + \sigma_\ell dW_t, \tag{2}$$

where μ_ℓ, σ_ℓ are the drift and volatility parameters and W is a standard Brownian motion process defined on a probability space (Ω, F, P), initialized at $LTV_0 \leq b$.

Without making some further assumptions, it would be hard to say much definitive about the nature of the default barrier b, which indeed might be some arbitrarily complex function of other state variables and parameters. In order to gain greater specificity, then, we will assume that (1) the barrier is endogenously determined in an entirely rational fashion, while at the same time assuming that (2) the mortgage is of a very simple, idealized form.

As to the first point, we postulate that the borrower decides to default exactly when that act maximizes the value of his position, which consists of the loan together with the underlying asset, the building. This is intrinsically a quite complicated decision, since for instance, account needs to be taken of the fact that current default forgoes the opportunity of defaulting in the future. One has to work back from all possible futures in order to correctly evaluate the current value of the mortgage, and as is well known, the resulting problem can be expressed as the solution of a backward partial differential equation (PDE), being a function of the value of the building asset.

Therefore, to simplify matters in the manner of our second point, and so focus attention on LTV, we suppress the role of these other variables by assuming that the borrower regards the contract as a perpetual, non-amortizing loan with continual constant payments, one that is within an environment of constant interest rates. This borrower, however, retains the right to default on the loan by turning over the property, in which case the lender is to have no further recourse.

In this idealized situation, the problem can be expressed in terms of a simpler, ordinary differential equation (ODE):

$$1/2\sigma^2 V^2 F_{vv} + (r-s)VF_v - rF + aL = 0, \tag{3}$$

along with the boundary condition

$$F_v(\hat{V}) = 1 \; when \; F(\hat{V}) = \hat{V}, \tag{4}$$

where F is the value of the mortgage and V represents the value of the underlying building asset, which is assumed to follow a geometric Brownian process,

$$\frac{dV_t}{V_t} = (\mu - s)dt + \sigma dW_t, \tag{5}$$

for service flow s, so that the loan-to-value ratio, $LTV_t \equiv L/V_t,$ for loan size L, indeed follows a geometric Brownian process, as claimed above.[1]

By a modification of the argument in Black & Cox (1976) the ODE (3) admits a solution for the optimal barrier of the form (see Kau and Keenan (1999))

[1]The geometric Brownian or lognormal process is by far the most popular process for modelling the evolution of an asset, both because of its simplicity and because of its assured nonnegativity.

$$\widehat{LTV} = L/\hat{V} = \frac{1+\beta}{\beta}\frac{r}{a} \equiv k\frac{r}{a}, \tag{6}$$

where r is the spot rate, $\beta = \frac{2r}{\sigma^2}$, a is the contract rate, and $\omega = -\beta$ is the negative root of

$$1/2\sigma^2\omega^2 + ((r-s) - 1/2\sigma^2)\omega - r = 0. \tag{7}$$

Since we have eliminated any role of time or a varying term structure by fiat, these elements are of course now absent, but more interestingly, we then also obtain that loan size does not appear individually, nor does original LTV_0; the borrower cares only about current loan-to-value LTV_t. Thus, in determining the critical $\widehat{LTV}$ that triggers default, it is of no consequence whether the loan was originally a 95% LTV loan for \$ 900,000 or an 80% LTV loan for \$ 200,000 – that is all in the past; given the same contract rates, their critical $LTVs$ will be one and the same. Now, being such different loans, it is likely that their contract rates will have in fact have been set differently, but such considerations are also in the past; the barrier determination simply takes this as given. Thus our modelling is in sharp contrast to typical reduced-form proportional hazard models, which generally put great emphasis on such easily obtained covariates as loan size, original LTV, and points. Note also that in the classical fashion of arbitrage reasoning, and unlike its associated volatility, σ_ℓ, which does have an influence, the mean, μ_ℓ, of the LTV process does not affect the barrier value b, though it does reenter in the calculation of the probability of default, which depends not only on the risk-neutralized LTV process that determines the barrier but also on the evolution of the actual LTV process. It is interesting to note that while we have eliminated prepayment by assumption, this is without loss of generality given the strict logic of our model, since Black & Cox (1976) show that, in the assumed environment, a borrower will always default before he would prepay. Commercial mortgages are, in any case, typically regarded as substantially less sensitive to prepayment than single-family mortgages, often having prepayment lockouts and/or substantial penalties.

As (6) shows, the barrier b may be thought of as being proportional to the contract rate and inversely proportional to the interest rate, much as with the classical formula for the value of a consol. Considering a *cluster* to consist of all loans in a region of one particular property type, we then want to infer the appropriate proportionality factor k for that cluster, and so use this to predict the barriers for all loans in that cluster.

Now, unlike original LTV_0, current LTV_t is not directly observable for a single loan, but we can construct an implied LTV for each loan by using the available REIT index for that property type to impute the evolution of the loan's LTV after origination. We calculated the implied LTV as

$$ILTV_t = \frac{current\_balance_t}{implied\_value_t} \quad \text{where} \quad implied\_value_t = \left(1 + \Delta reit_t\right) implied\_value_{t-1},$$

with $reit_t$ being the property-type REIT index and $implied\_value_0 = L/LTV_0$. Capozza *et al.* (1997), as well as Haurin *et al.* (1991), discuss alternate ways of inferring current LTV. Given the various critical $\widehat{LTV}s$ of the defaulted loans in a cluster, which coincide with their implied $LTVs$ at time of default, as well the interest rate at such time and the contract rate for each loan, we can use (6) to calculate the implied proportionality constants, loan by loan. The average of these values then yields the cluster proportionality factor k we seek that best explains the observed defaults for this cluster. Given the cluster proportionality constant, we can then again use (6) to calculate the barrier of each and every loan in that cluster.

The resulting proportionality constant is somewhat biased downward, since no account was taken of the observations for which default did not occur and the critical $\widehat{LTV}s$ not encountered, which are typically associated with higher critical $\widehat{LTV}$ values. Accounting for such censoring is, however, more statistically sophisticated, and we wished to emphasize the simplest use of the current model. A procedure even more faithful to the assumed nature of the default would be to decompose β according to (7) and use this to back out the service flows. As well as sacrificing simplicity, however, this overspecifies the estimation procedure and tends to yield unreliable estimates; better results are obtained by using theory to guide the general functional form of the barrier, but then simply using the data to estimate β. In any case, for the parameter values used, β in equation (6) and (7) is rather insensitive to the interest rate.

Given its barrier, the default probability for a loan can then be written as follows:

$$P(\tau \leq t) = P\left(\max_{0 \leq s \leq t} LTV_s \geq b\right)$$

$$= \Phi\left(\frac{-\ln(\frac{b}{LTV_0}) + (\mu_\ell - \frac{\sigma_\ell^2}{2})t}{\sigma_\ell \sqrt{t}}\right) + e^{\frac{2\left(\mu_\ell - \frac{\sigma_\ell^2}{2}\right)\ln\left(\frac{b}{LTV_0}\right)}{\sigma_\ell^2}} \Phi\left(\frac{-\ln\left(\frac{b}{LTV_0}\right) - \left(\mu_\ell - \frac{\sigma_\ell^2}{2}\right)t}{\sigma_\ell \sqrt{t}}\right), \qquad (8)$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution. To find this default probability, the drift and volatility parameters, μ_ℓ and σ_ℓ, of the process in equation (2) need to be calculated. This is, however, straightforward since the log return of the process is an independently drawn random variable from a normal distribution,

$$\ln\left(\frac{LTV_t}{LTV_{t-1}}\right) \sim N\left[\left(\mu_\ell - \frac{\sigma_\ell^2}{2}\right), \sigma_\ell^2\right]. \tag{9}$$

Again using REIT indices, but now by region and type, sample moments of $\ln(\frac{LTV_t}{LTV_{t-1}})$ are calculated to find μ_ℓ and σ_ℓ for each such region and property type in our sample.

Estimation

Data

The commercial loan data set was provided by WOTN, LLC, a risk management software firm. Interest rates are from the Federal Reserve Board's web site. REIT indices are from the National Council of Real Estate Investment Fiduciaries (NCREIF) and Bloomberg. CMBS loan prices, deal structure and loan characteristics are from Trepp's comprehensive historical commercial loan database.[2]

Table 1 lists the number of loans for every property type and region, whereas Table 2 expresses defaults as percents of the total number of loans. The data consists of $50,099$ fixed rate CMBS loans observed at monthly intervals from June 1998 to May 2005, with originations starting in 1995.[3] There were $1,051$ defaulted loans in the sample, which is 2.10% of the entire population. All of the observations are censored or truncated, so great care would be required if one were instead to use standard duration techniques to calculate the default rates. (Yildirim (2007) shows how to statistically estimate such parameters in the case

[2]Trepp, LLC is a leading provider of CMBS and commercial mortgage information.
[3]Earlier observations from 1995 to 1998 were considered to be unreliable.

of such heavy censoring, using a mixture model.)

Results

In Figure 2, we report the means and volatilities for the various LTV processes, distinguished by property type and region. It will be observed that property type is far more important in determining the process than is region. When averaged over all property types, the mean drift turns out to be -.01267, while the standard deviation is .05031.

In Table 3, we report our main results concerning the predicted probabilities of default for the various loans. Note that the probability of default for a loan varies not just by its property type and region, but by its contract rate, its time to maturity, and by the interest rate at the time of origination. Thus, in Table 3 we obtain a distribution of probabilities of default over the various different loans of a given type and within a region, as indicated by having entered both the mean and standard deviation of that distribution in each cell. In Figure 3, we display two examples of the resulting distributions of probabilities.

It is interesting to compare the estimated probabilities in Table 3 to the observed ones in Table 2, though one must be careful with such comparisons. For one thing, whether in terms of observed or predicted probabilities, the resulting probabilities of default are typically rather small, whereas the sample size per cluster is not often correspondingly large, so that even were the predicted probabilities exactly correct, one would sill have to expect noticeable differences between the predicted and the observed probabilities of a particular cluster. Obviously, the comparisons work better for totals of a region or a property type than for the individual clusters, and so, we will concentrate our remarks on these totals. One generally expects the predicted probabilities to be higher than the observed ones, since they are calculated over the entire term to maturity of a loan, whereas Table 2 merely records the number of defaults over the observed window.[4] This expectation is certainly confirmed in general, as may be seen by comparing overall predicted default (2.4%) to overall experienced default (2.1%), though there are a number of exceptions to this within property-type and regional categories, of which the Lodging sector is perhaps the most prominent.

The obvious explanation for the larger of the mispredictions is that the populations within the constituent clusters are somewhat heterogenous. This can be easily used to explain why predicted probabilities are abnormally low, as is the case with the Pacific region. If there is a subgroup that defaults at unusually low $LTVs$, this will then result in one overpredicting the probability of default for the general population of the cluster, particularly if the possibly large remainder of the population is hardly defaulting at all, and so making little contribution to the predicted barrier value. This argument is a bit weaker in reverse, though, when one needs to account for underpredictions. It requires that there be a substantial subpopulation that is defaulting at a noticeable rate, together with a much smaller subpopulation highly prone to default. The predicted barrier will then be near the average of the two barriers, but because the probability of default increases at an increasing rate with a lower barrier value, this average barrier value won't produce as much default as is actually observed. Thus, the resulting explanation of overprediction works best when the observed default levels are smaller, while the offered explanation of underprediction works better when there is more substantial default overall, and where the latter has difficulty explaining as much disparity as can the former. It is of some comfort, then, that the argued-for patterns are

[4]The degree by which the former ought to exceed the latter will vary across clusters, however, according to such features as the times to maturity of the loans within the cluster and their estimated LTV process, as well as being affected by such issues as when the loans originated in comparison to the observation window.

borne out in the more prominent cases of overprediction and underprediction observed between Tables 2 and 3.

Conclusion

One way to distinguish a reduced-form from a structural model, is to note that no feasible reduced-form model based on actual data is likely to assign much probability to a particular loan defaulting in a particular period, whatever the covariates happen to then be, since few loans ever do default in a given period, and reduced-form models seldom distinguish greatly among loans, to a large extent regarding which among a group of loans actually defaults as being a random, unexplained event. On the other hand, with a structural model such as the current one, under the appropriate circumstances it becomes inevitable that a default will occur for a particular loan, i.e., it occurs when current LTV is sufficiently high. One can further identify our structural model as being truly endogenous by the fact that the resulting default barrier is not simply externally imposed, but is instead determined from within the model by the rational considerations of the borrower acting to minimize the financial burden of the loan.

Using our model, we have shown a rather straightforward procedure by which one can predict the probability of eventual default, beginning at the origination of the loan, the time when a lender would be most interested in making such a determination. The simplicity of our procedure arises from taking that variable, the current loan-to-value (LTV) ratio, well-recognized to be the single most important dynamic determinant of default behavior, and concentrating all our efforts on modelling its consequences for default. While founded on an optimizing structural model, our estimation procedure is nonetheless sensitive to actual default experience, not only in determining the underlying LTV process appropriate for a loan, but also in determining that endogenous boundary, whose passage by the loan's current LTV will indicate that the borrower has rationally entered into default.

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Table 1:
There are 50,099 fixed rate loans from June 1998 to May 2005, of which 1,051 loans defaulted.

	Industrial	Lodging	Multifamily	Office	Retail	*Total*
Southeast	524	514	2423	781	2854	7096
Mideast	504	414	1266	917	2371	5472
Northeast	838	418	3454	1578	2415	8703
East North Central	430	276	1807	624	1962	5099
West North Central	155	119	766	237	730	2007
Mountain	569	221	1439	646	1693	4568
Southwest	504	276	3000	592	2261	6633
Pacific	2014	521	2995	1982	3009	10521
Total	5538	2759	17150	7357	17295	50099

Table 2:
The percentage of loans actually defaulting, for each property type by region

	Industrial	Lodging	Multifamily	Office	Retail	*Total*
Southeast	2.48%	13.04%	1.86%	1.15%	2.59%	2.93%
Mideast	1.19%	11.84%	2.45%	0.76%	2.83%	2.92%
Northeast	1.55%	3.11%	0.61%	1.14%	1.53%	1.17%
East North Central	4.19%	11.59%	1.16%	4.49%	2.70%	2.98%
West North Central	1.94%	16.81%	0.78%	2.53%	2.47%	2.64%
Mountain	1.58%	14.48%	1.04%	1.55%	1.12%	1.86%
Southwest	1.59%	14.49%	3.53%	2.70%	2.74%	3.50%
Pacific	0.30%	3.26%	0.13%	0.55%	0.70%	0.56%
Total	1.37%	9.79%	1.45%	1.43%	2.03%	2.10%

Table 3: Estimated default probabilities, averaged over all loans, for each property type and region. Average standard deviations are given in parenthesis.

	Industrial	Lodging	Multifamily	Office	Retail	*Total*
Southeast	2.74%	6.65%	0.88%	1.23%	3.18%	2.94%
	(0.1493)	(0.3006)	(0.0852)	(0.1176)	(0.2790)	(0.1863)
Mideast	1.88%	9.81%	0.97%	1.23%	0.79%	2.94%
	(0.0724)	(0.4116)	(0.0835)	(0.1296)	(0.1613)	(0.1717)
Northeast	1.30%	4.69%	1.33%	1.34%	1.39%	2.01%
	(0.0894)	(0.1993)	(0.0899)	(0.1009)	(0.1064)	(0.1172)
East North Central	0.59%	3.51%	0.57%	1.75%	3.39%	1.96%
	(0.0333)	(0.1278)	(0.0613)	(0.1408)	(0.2380)	(0.1203)
West North Central	0.15%	5.96%	0.67%	3.23%	1.52%	2.30%
	(0.0065)	(0.2476)	(0.0824)	(0.2174)	(0.1709)	(0.1450)
Mountain	1.75%	3.58%	2.14%	0.90%	0.59%	1.79%
	(0.1338)	(0.1772)	(0.1321)	(0.0595)	(0.0761)	(0.1157)
Southwest	1.08%	7.16%	1.00%	2.59%	3.06%	2.98%
	(0.0926)	(0.2866)	(0.0856)	(0.1764)	(0.2940)	(0.1870)
Pacific	1.21%	2.63%	6.04%	1.10%	0.52%	2.30%
	(0.0870)	(0.1112)	(0.1734)	(0.0704)	(0.0535)	(0.0991)
Total	1.34%	5.50%	1.70%	1.67%	1.81%	2.40%
	(0.0830)	(0.2327)	(0.0992)	(0.1266)	(0.1724)	(0.1428)

Figure 1: First passage time model: Simulated geometric Brownian motion initiated at V_0 with barrier b.

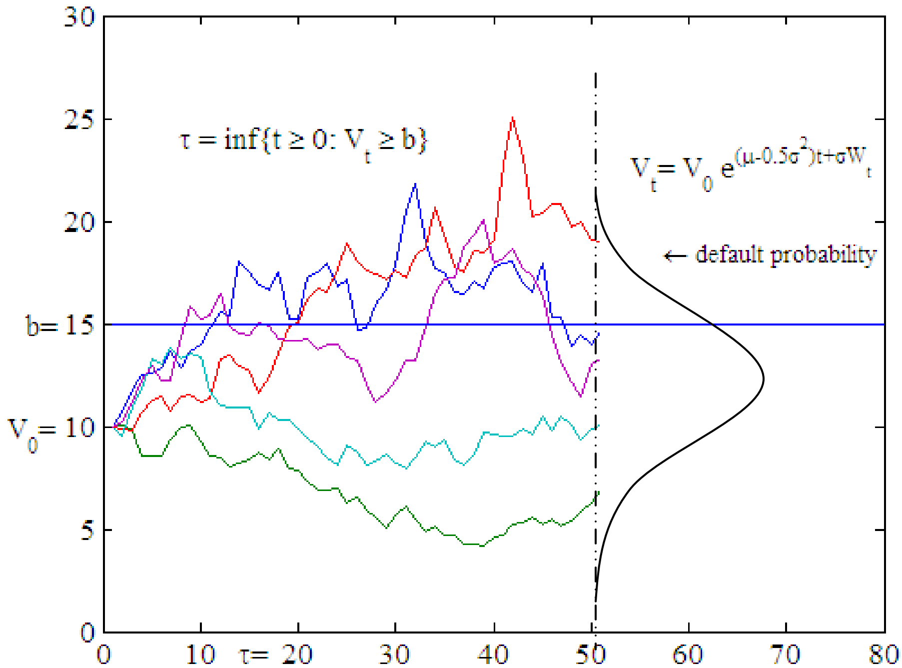
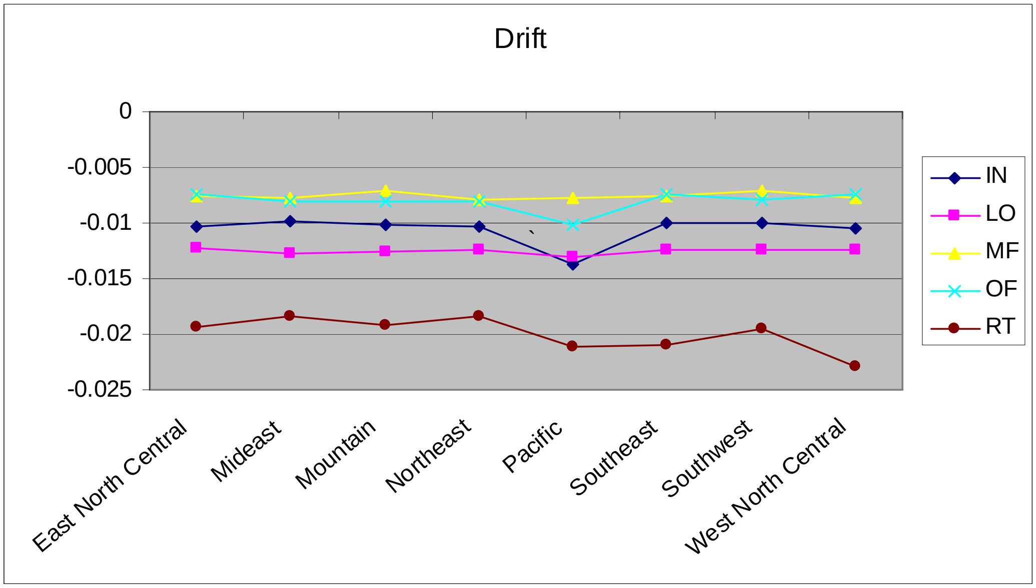


Figure 2: Drifts and volatilities of the ILTV processes are given in below. Average drift and volatilities are -.01267 and 0.05031 respectively.



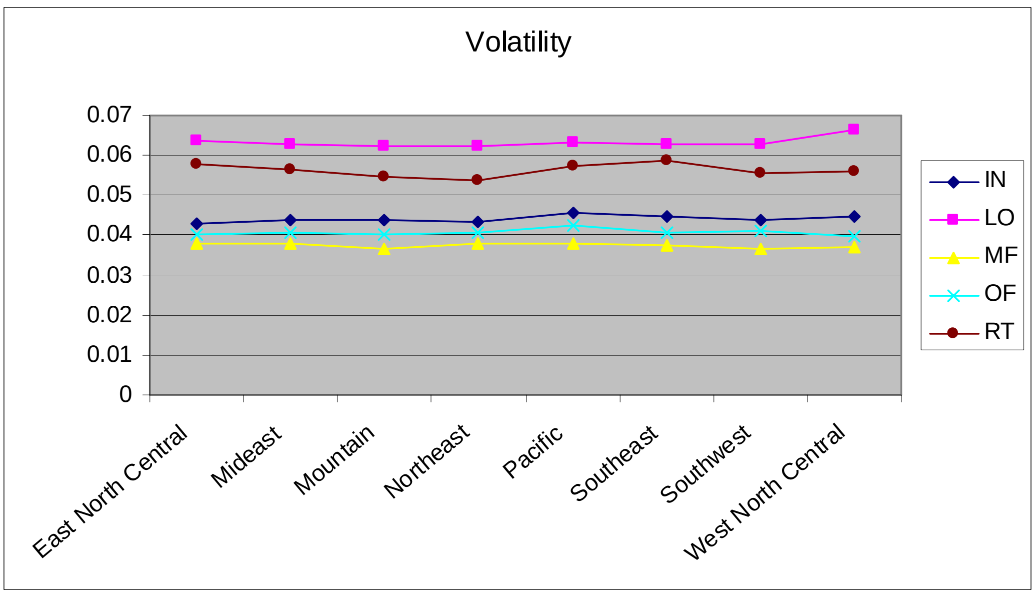
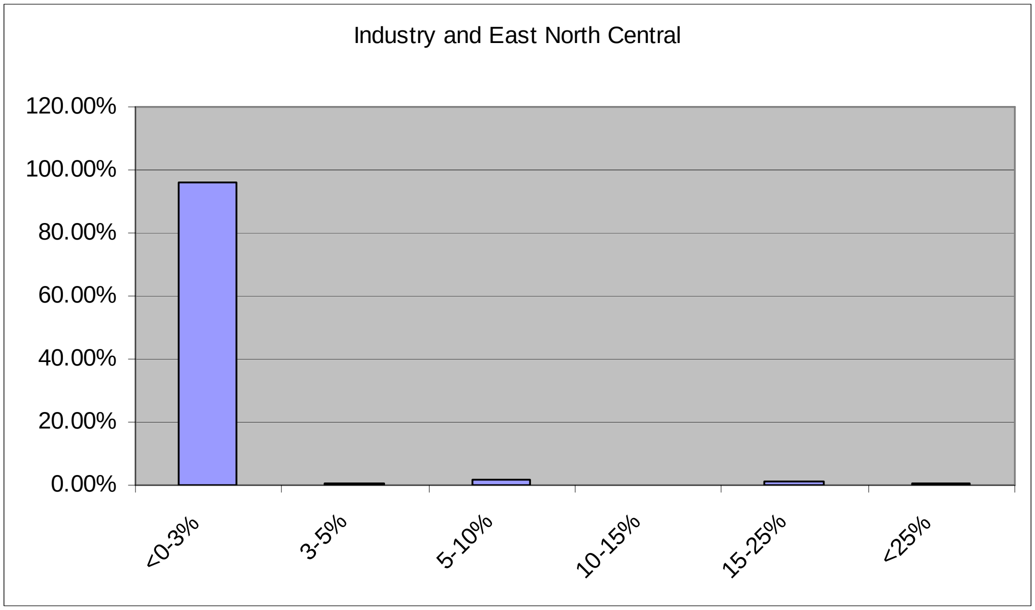


Figure 3:

Panel (A)
Distribution of the estimated probability of default among Industrial loans in the East North Central Region



Panel (B)
Distribution of the estimated probability of default among Lodging loans in the Pacific Region