How Valuable is Credit Card Lending?

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Credit card loans are a growing component of earnings and market value for banks. The valuation of credit card loans, however, has received surprisingly little attention. This article develops and estimates an arbitrage-free model for valuing portfolios of credit card loans. The model uses Federal Reserve quarterly call report data for the credit card portfolio balances, rates earned, and losses and recoveries. Interest rates are fitted using a one-factor model with deterministic volatilities.

The valuation estimates show that $1.00 in a credit card loan is worth about $1.25 to the issuing bank. These estimates are consistent with the percentage premiums obtained in recorded sales of credit card portfolios.

Over the past 30 years, credit cards have evolved to become a major source of financing for consumer transactions. In 1970, credit card-related consumer debt totaled $2 billion. By 2000 it had grown to $626 billion. Ausubel [1997] estimates that in 1995 the average credit card balance per household was $4,400 (including balances that are quickly repaid and thus do not incur interest charges). Durkin [2000] reports that in 1995 two-thirds of U.S. households held one or more credit cards and carried an average balance—after the most recent payment—of $3,160.[1]

The growth rate of credit card use remains high. Recent estimates in the popular press suggest that average household balances have risen to $8,123.[2]

Our objective is to develop a tractable methodology for pricing and hedging portfolios of credit card loans. This analysis is of interest for three reasons. First, financial and non-financial corporations are paying increasingly close attention to the profitable opportunities in consumer credit. A good example is Sears, Roebuck, which established Sears National Bank, a special-purpose bank to facilitate its credit card operations. By 1999, 56% of Sears' operating profits came from these operations. At that time, Sears earned more money from financing than from selling the underlying products.

Bassett and Zakrajsek [2000] show that credit card loans have been much more profitable for banks than commercial loans and leases. In 1995, banks earned a 10.02% spread on credit card operations, dominating the 3.37% spread for commercial loans and leases. While credit card loans represented 4.7% of bank assets, they generated 17.0% of bank profits. Similarly, Ausubel [1997] reports that, from 1983 to 1993, the return on assets from credit card loans was roughly four times the overall ROA.

The second reason our study is useful is that there is scant research on valuing credit card loan portfolios. This is surprising, given the importance of this sector to the banking industry. The academic literature on credit cards has focused on the competitive structure of the credit card industry rather than valuation (see Ausubel [1991, 1997, 1999], Brito and Hartley [1995]), and Calem and Mester [1995]. Kahn, Pennacchi, and Sopranzetti [2002] provide a similar structural analysis of non-credit card consumer loans.

Our focus is fundamentally different. We use an asset pricing perspective to develop the first tractable valuation model for credit card loan portfolios. Relying on Jarrow and van Deventer [1998] and Janosi, Jarrow, and Zullo [1999], we develop an arbitrage-free valuation approach that models the evolution of credit card loan portfolio balances, rates earned, and losses.

Finally, the issue of managing credit risk is now receiving much greater attention. For example, the Bank for International Settlements is currently revising its regulatory requirements for credit risk. A concept release states that "banks should now have a keen awareness of the need to identify, measure, monitor, and control credit risk." In 2003, the BIS is expected to issue a new set of standards that allow banks to use internal models for credit risk management.[3]

One contribution of our work is a pricing model that banks can use to measure and manage the risks of their credit card operations. The risk management applications follow from the relationship of pricing and hedging. Hedging typically requires creating a synthetic exposure opposite to the underlying exposure. Creating the synthetic exposure, however, requires a well-defined pricing model.

We implement our pricing model using quarterly call report data from 1984 though 1995. We provide parameter estimates for a typical bank, based on averages from our data, and for five individual banks. Our results show the model does a good job of describing the aggregate data. During our sample period, the estimated net present value of a credit card portfolio for the average bank was 25%. That is, $1.00 in a credit card loan was worth about $1.25. These findings are consistent with Ausubel [1991], who reports that credit card portfolios are sold at roughly a 20% premium.

I. THE MODEL

Our model is based on Jarrow and van Deventer [1998] and Janosi, Jarrow, and Zullo [1999]. We extend these models to incorporate the characteristics of the credit card market. Traded are default-free zero-coupon bonds and a money market account whose value at date t is $B(t)$. Let $[0, \tau]$ be the trading horizon.[4]

Let $P(t, T)$ be the time t price of a default-free zero paying a sure dollar at time T. Continuously compounded forward rates $f(t, T)$ are implicitly defined by

$$P(t, T) = \exp\left(-\int_t^T f(t, s)ds\right) \tag{1}$$

The money market account earns interest at the spot rate $r(t) = f(t, t)$:

$$B(t) = \exp\left(\int_0^t r(s)ds\right) \tag{2}$$

Outstanding credit card portfolio balances at time t are denoted by $L(t)$, and the credit card rate earned is denoted by $i(t)$. The rate $i(t)$ includes the non-interest costs of servicing the accounts. We define $l(t)$ as the percentage losses on the credit card loans due to defaults adjusted by any recoveries received.

Using no arbitrage and complete markets, Jarrow and van Deventer [1998] value the credit card loan as an interest rate derivative. They show that the net present value of the credit card loan, $V(0)$, can be written as:

$$V(0) = \tilde{E}_0\left(\int_0^\tau \frac{L(t)[i(t)-l(t)]}{B(t)}dt\right) - \tilde{E}_0\left(\int_0^\tau \frac{L(t)r(t)}{B(t)}dt\right) \tag{3}$$

where $\tilde{E}_0(\ .\ )$ is the time 0 expectation using an equivalent martingale measure $\tilde{Q}$.

Equation (3) has a useful economic interpretation. The first integral represents the present value of the benefits because $L(t)[i(t) - l(t)]$ is the time t cash inflow from the loans. These cash inflows are discounted at $B(t)$ and summed across time, and an expectation is taken. The second integral represents the time t costs $L(t)r(t)$ from financing the loans, aggregated and discounted to the present.

As Equation (3) indicates, credit card portfolios exhibit three risks:

1. Risks due to stochastic interest rates [represented by $r(t)$].
2. Risks due to fluctuations in credit card balances [represented by $L(t)$].
3. Risks in credit card returns—inflows minus losses [represented by $i(t) - l(t)$].

Equation (3) explicitly incorporates all three risks.

Note it imposes no substantive restrictions on the stochastic processes for interest rates, credit card loan balances, or credit card net interest rates earned. The only restriction imposed on these evolutions is that they depend on (at most) a finite number of common factors; see Jarrow and van Deventer [1998] for further clarification.[5] These common factors could be thought of as representing the fundamental economic forces driving the macroeconomy and the banking sector.

Term Structure Evolution

As a first pass for the analysis, we use a one-factor term structure model with deterministic volatilities (see Heath, Jarrow, and Morton [1992]). This model has proven useful in many applications both because it provides a reasonable empirical fit to spot interest rate movements and because its structure facilitates closed-form solutions and lattice computations (see James and Webber [2000]). This model is sometimes known as the extended Vasicek model. The analysis is easily generalized to a multiple-factor model.

Following Janosi, Jarrow, and Zullo [1999], we assume that the term structure process is described by the evolution of the spot rate of interest under the martingale measure $\tilde{Q}$:

$$dr(t) = a[\bar{r}(t) - r(t)]dt + \sigma d\tilde{W}(t) \qquad (4)$$

where $a \neq 0$, $\bar{r}(t)$ is a deterministic function of t, $\sigma > 0$, and $\tilde{W}(t)$ is a standard Brownian motion under $\tilde{Q}$ initialized at $\tilde{W}(0) = 0$.

The spot rate of interest follows a mean-reverting process under the martingale measure. It has a mean-reverting parameter a, a long-run spot rate of $\bar{r}(t)$, and a volatility of σ. As shown in Heath, Jarrow, and Morton [1992], to match an arbitrary initial forward rate curve $\{f(0, t) \text{ for } t \in [0, \tau]\}$, one must set the long-run spot rate equal to

$$\bar{r}(t) = f(0, t) + \left(\partial f(0, t)/\partial t + \sigma^2\left(1 - e^{-2at}\right)/2a\right)/a \qquad (5)$$

We can rewrite Equation (4) as:

$$r(t) = f(0, t) + \sigma^2\left(e^{-at} - 1\right)^2/2a^2 + \int_0^t \sigma e^{-a(t-s)} d\tilde{W}(s) \qquad (6)$$

Equation (6) makes explicit the restriction that there is only one (independent) macroeconomic factor generating interest rate movements. This *unobservable* macroeconomic factor corresponds one-to-one with the spot rate of interest. The spot rate of interest, therefore, becomes an *observable* proxy for its fluctuations. This macroeconomic factor also influences both credit card loan balances and net credit card rates earned, including credit card losses.

Credit Card Loan Balance Evolution

We can use the one-to-one correspondence between the macroeconomic factor and the spot rate of interest to model the evolution of both credit card loan balances and the credit card rates earned.

For empirical implementation, following Janosi, Jarrow, and Zullo [1999], we assume that credit card loan balances follow the stochastic process given by:

$$\log L(t) = \log L_0 + \mu t + (1 - \gamma^t)\log c + \int_0^t \alpha_{t-s} r(s) ds \qquad (7)$$

where $c > 0$, $L_0 > 0$, $0 \leq \gamma < 1$, the μ are constants, and α_{t-s} is a deterministic function of $t - s \geq 0$.

This process for credit card loan balances is selected because of its analytic simplicity and its ability to match the historical time series evolution of credit card balances. It is constructed to exhibit growth toward a long-run steady-state process. Indeed, credit card loan balances start at the initial balances and are assumed to grow in an exponential manner at the rate μ.[6]

Two effects adjust this growth rate. The first is movement toward the long-run or steady-state credit card balances of cL_0, where c is the percentage of the current credit card balances that the steady-state balances represent.[7] This adjustment to the long-run balances is captured by the term $(1 - \gamma^t) \log c$. As t increases, given $\gamma < 1$, this term approaches $\log c$. We combine this with $\log L_0$ to obtain the long-run balances. If the bank's credit card loans are expected to increase in the long run, then $c \geq 1$, which means the current balances are lower than the steady-state balances.

The second adjustment in the growth rate is due to the average level of interest rates, represented by the last term in Equation (7). Changing interest rates influence credit card balances in at least two ways. One, unpaid credit card balances grow at a rate determined by the spot rate.

Two, depending on the history of interest rates, unpaid balances may be repaid more or less quickly. For example, as rates increase, the cost of borrowing is high, and borrowers have an incentive to repay. Also, as rates increase, the health of the economy may suffer, and borrowers may be less able to repay. Hence, the sign of α_{t-s} is ambiguous (and must be determined empirically).

For empirical estimation, we need to discretize Equation (7). We use a discretization from Janosi, Jarrow, and Zullo [1999]. Let Δ be a fixed time interval, the time interval between data observations measured in years (say, $\Delta = 1/4$, or three months). The regression index will be denoted by $j = 0, 1, 2, \ldots$. The correspondence between time and the regression index is that $t = j\Delta$. Define the average spot rate $R(t - j\Delta)$ over the time interval $[t - (j + 1)\Delta, t - j\Delta]$ as:

$$R(t - j\Delta)\Delta = \int_{t-(j+1)\Delta}^{t-j\Delta} r(s)ds \quad \text{for } t \geq \Delta$$
$$j = 0, 1, \cdots, N - 1 \text{ where } N\Delta = t \qquad (8)$$

Assuming that α_{t-s} is piecewise constant, the discretized credit card loan portfolio evolution simplifies to:[8]

$$\begin{aligned}\log L(j\Delta) = {} & \gamma^{\Delta}\log L((j-1)\Delta) + \left(1 - \gamma^{\Delta}\right)\log cL_0 + \\ & \gamma^{\Delta}\mu\Delta + \mu\left(1 - \gamma^{\Delta}\right)j\Delta + \\ & \alpha\Delta(1 - \gamma^{\Delta})R(j\Delta) + x_j \qquad \text{for } t \geq 1 \qquad (9)\end{aligned}$$

where $\alpha = \sum_{j=-N+1}^{0}\alpha_j$ is a constant measuring the aggregate interest rate sensitivity of credit card loan balances; the error term is $x_0 \equiv 0$ with $x_j = \rho x_{j-1} + \tilde{x}_j$; and the $\tilde{x}_j$ are independent and identically distributed (iid) normal $(0, \sigma_{\tilde{x}}^2)$ for $j \geq 1$. In this specification, the error terms can be autocorrelated.

Credit Card Net Interest Rate Evolution

The net credit card rate $n(t) \equiv [i(t) - l(t)]$ is assumed to satisfy a stochastic process similar to that satisfied by the logarithm of the credit card balances $\log[L(t)]$. Hence, we let

$$n(t) = k + \pi^t\left(n_0 - k\right) + \int_0^t \beta_{t-s}r(s)ds \qquad (10)$$

where k, n_0, and $1 > \pi \geq 0$ are constants, and β_{t-s} is a deterministic function of $t - s \geq 0$.

In Equation (10), k represents the steady-state floor on the net credit card rate earned.[9] $1 > \pi \geq 0$ measures the rate of decay of the current net credit card rate earned n_0 to the long-run floor rate k. The function β_{t-s} measures the sensitivity of deposit rates to the average level of spot interest rates $r(t)$. It reflects the impact of spot rates on the rate charged $[i(t)]$ and the losses incurred $[l(t)]$.

The sign of β_{t-s} is ambiguous a priori. Indeed, as the average rate rises, one would expect both $i(t)$ and $l(t)$ to increase, but it is the differential growth rate that determines the net. This differential could be positive or negative, depending on the bank-specific credit card borrower pool and the changing economic conditions as the spot rate increases.

As before, to implement Equation (10) empirically, we need to discretize the integral. Following Janosi, Jarrow, and Zullo [1999], the simplified equation is:

$$\begin{aligned}n(j\Delta) = {} & \pi^{\Delta}n((j-1)\Delta) + (1 - \pi^{\Delta})k + \\ & \beta\Delta(1 - \pi^{\Delta})R(j\Delta) + z_j \qquad \text{for } j \geq 1 \qquad (11)\end{aligned}$$

where β is a constant measuring the cumulative interest rate sensitivity of the net credit card rate charged; and the error term is $z_0 \equiv 0$; the z_j satisfy $z_j = \rho z_{j-1} + \tilde{z}_j$; and the $\tilde{z}_j$ are iid normal $(0, \sigma_{\tilde{z}}^2)$ for $j \geq 1$.

Simplified Credit Card Loan Portfolio Valuation

Given the stochastic evolutions for the term structure of interest rates, credit card loan balances, and net credit card rates as detailed above, the credit card valuation Equation (3) can be evaluated in closed form. The expression for the net present value of the credit card loans is:[10]

$$\hat{V}(0) = e^{[\sigma_x^2/(1-\gamma^{2\Delta})]} \times cL_0\int_0^{\tau}(1/c)^{\gamma^t} e^{\mu t+\mu_1(t)+\sigma_1^2(t)/2} \times \begin{pmatrix} -[\mu_3(t) + \sigma_{13}(t)] + \\ [k + \pi^t(n_0 - k) + \\ \mu_2(t) + \sigma_{12}(t)] \end{pmatrix} dt \qquad (12)$$

where σ_x^2 is the variance of the error term in the credit card loan evolution regression (9); and the expressions for $\mu_1(t)$, $\mu_2(t)$, $\mu_3(t)$, $\sigma_1^2(t)$, and $\sigma_{13}(t)$ are deterministic functions of time involving the spot rate evolution's parameters and the interest rate sensitivity parameters (α, β), whose explicit formulas can be found in the appendix.

For comparison purposes across different credit card loan portfolios and time, we express the net present value

EXHIBIT 1
Descriptive Statistics

Bank	Data Range[a]	Total Assets[b]	Credit Card Balances[b]	Merger Information 1984-1995
Chase Manhattan	1984:2 - 1995:4 N = 47	1,137,765	559,239	No direct acquisitions
Vermont	1984:2 - 1995:4 N = 47	388,925	7,779	Acquired Valley Bank, White River Junction, VT, on 9/14/91
Bank of Delaware	1984:2 - 1995:4 N = 47	1,113,716	33,315	None
First USA	1985:3 - 1995:4 N = 42	439,576	419,064	None
MBNA	1991:2 - 1995:4 N = 20	4,926,272	2,995,454	None

[a]*The 1984:12-30 data point is deleted due to a missing observation in the Treasury bond data set.*

[b]*These figures are in thousands, and they correspond to the first sample set for each bank.*

of credit card loans $V(0)$ in percentage terms; that is, $V(0)/L_0$.

II. DESCRIPTION OF THE DATA

There are two types of data: 1) the bank's credit card loan balances and net interest rates earned, and 2) the Treasury bond data.

The credit card loan information is obtained from the quarterly Federal Reserve Bank Call Reports from 1984:2 through 1995:4.[11] Five banks are selected for investigation: Chase Manhattan, Vermont National Bank, Bank of Delaware, First USA, and MBNA. These five banks are chosen because of diversity in their asset sizes and credit card operations. First USA and MBNA are banks that specialize in credit card lending.

Exhibit 1 provides descriptive statistics for the banks for the various time periods considered. Note that both First USA and MBNA started reporting data some time after the start of our data set. The second column gives the total assets of the banks considered on the first date in our sample set. Of the banks considered, in the second quarter of 1984, Vermont National Bank was the smallest, with total assets of $389 million, and Chase Manhattan the largest, with total assets of $1,138 million.

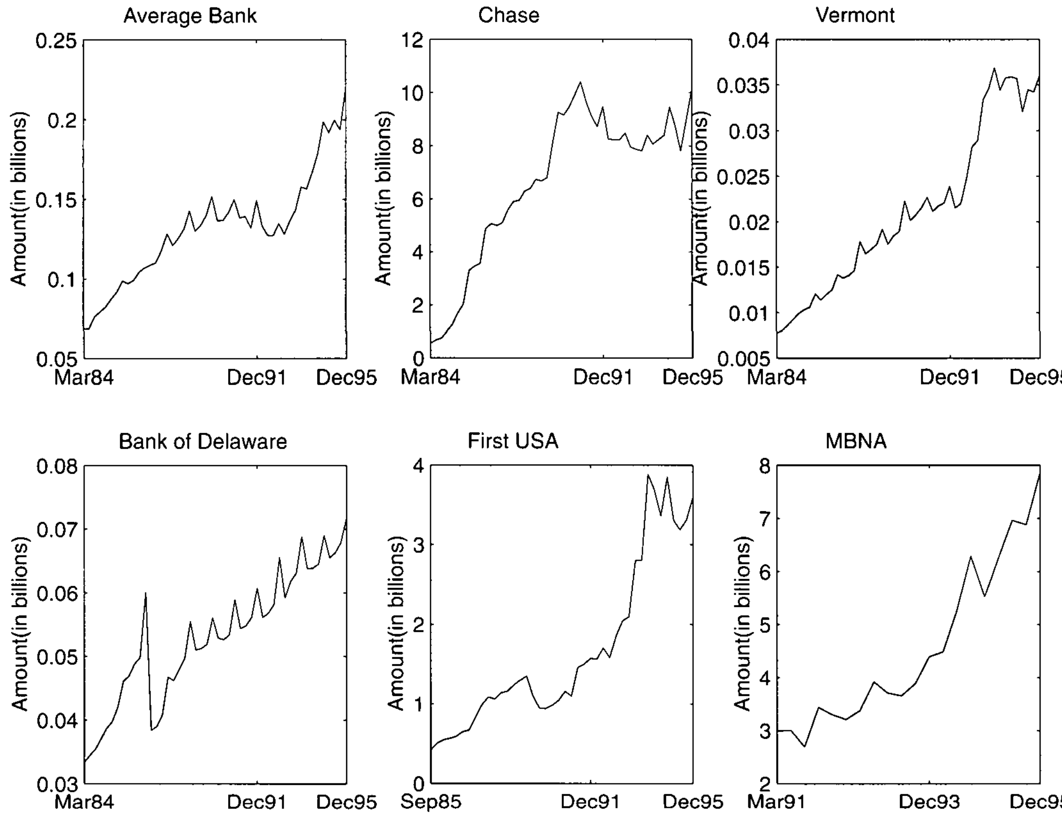
The amount of each bank's credit card balances is provided in the next column. At the start of our data set, credit card balances represented 49% of Chase Manhattan's total assets, only 2% of Vermont's, and 95% of First USA's.

Merger activity is indicated in the last column. None of the five banks selected for analysis exhibited significant merger activity over the sample period. The only direct merger occurred when Vermont National Bank acquired Valley Bank of White River Junction, Vermont, on September 14, 1991. First USA was acquired by Bank One, but this transaction occurred three years after our sample period. First USA did, however, change its name three times.[12] Chase Manhattan was also involved in acquisitions, but through its subsidiary, Chase Manhattan Bank NA. The operations of the subsidiary were formally merged with the parent in 1996, after the end of our sample period.

For comparison purposes, an average bank was formulated from all the banks included in the database that issued credit cards. The average banks' quarterly credit card balances, rates earned, charge-offs, and recoveries were computed by summing across all available credit card balances, rates earned, charge-offs, and recoveries in the data set each quarter, and then dividing by the total number of banks included.

At the start of our sample period, there were 595 banks included in the index. At the end of the sample period, there were 951 banks included. This increase in the number of

EXHIBIT 2
Monthly Credit Card Balances over Sample Period



banks issuing credit cards reflects both consolidations due to mergers and acquisitions as well as the increased popularity of credit card assets over the sample period.

For the purpose of our estimation, we define the variables:

$n(t) = i(t) - l(t)$ is the quarterly credit card income less charge-offs plus recoveries as a percentage of average balances over the quarter;

$L(t)$ is the average credit card loan balances over the quarter; and

$R(t)$ is the three-month forward rate.

These credit card quantities are constructed using the information in the call report data as follows:

Quarterly credit card income: The income from credit card operations. This includes interest changes on outstanding balances and transaction fees from merchants, but excludes annual fees from cardholders. Credit card income is recorded in a quarterly year-to-date format.

For each year, the variable $i(t)$ is measured as the quarterly change in the reported income from time $t-1$ to time t, divided by the average credit card loans outstanding computed over the same period. Multiplying by four annualizes these quarterly rates.

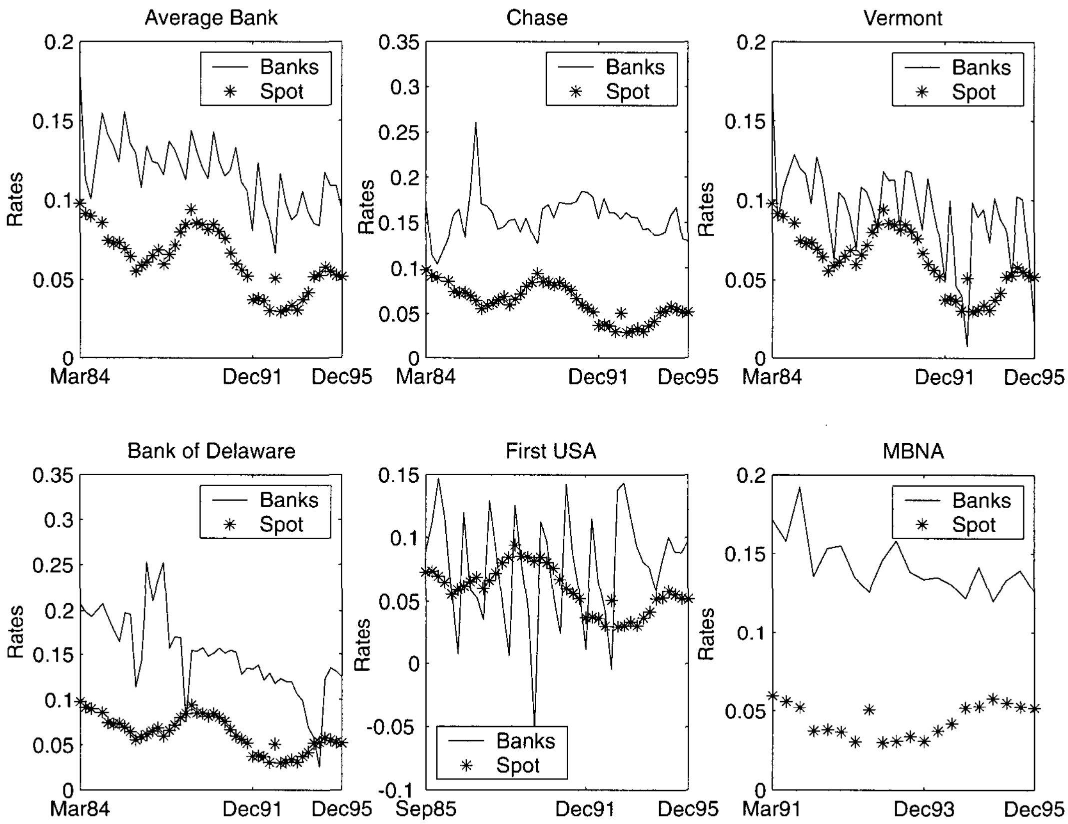
Quarterly charge-off: The accounting loss from credit card loan defaults. The variable $l(t)$ is measured as the level of credit card losses from time $t - 1$ to time t, divided by the average credit card loans outstanding computed over the same period.

Quarterly recoveries: The accounting recoveries from credit card loans. The recoveries are measured as the level of the reported recoveries from time $t-1$ to time t, divided by the average credit card loans outstanding computed over the same period. The recoveries are then subtracted from $l(t)$.

Credit card loans: The total amount of credit card loans issued by banks.[13]

The six graphs in Exhibit 2 plot the time series patterns of the various banks' credit card balances over the sample period. There was dramatic growth in credit card balances over the observation period, except during 1989-1992 when balances appeared to stabilize.

EXHIBIT 3
Net Credit Card Interest Earned



Banks – Annualized net interest rate earned on the credit card loan portfolio.
Spot – Annualized spot rate of interest.

Exhibit 3 provides the time series graphs of the banks' net interest rates earned on an annualized basis. Shown for comparison is the spot rate of interest. There appears to be some slight correlation between the spot rate and the net interest rates earned over the sample period. For most of the observation period, the net interest rate earned on the credit card loan portfolios (marginal benefit) exceeds the spot rate of interest (marginal cost) for each bank studied except for First USA. First USA's net interest rate earned fluctuates in a saw-tooth pattern both above and below the spot rate of interest.

The net interest rates earned on the various accounts exhibit seasonal patterns. These patterns could result for two reasons. First, they may be the result of seasonal borrowing demands (e.g., during holiday periods borrowers might let their credit card balances grow). Second, the patterns may result from reporting inaccuracies in the Call Report. In the first case, the seasonalities would represent real fluctuations in net interest rates earned, and thus become an important component to model in the data. In the second case, the seasonal patterns represent noise, and the series should be corrected for that pattern before use.[14]

Since we have no strong ex ante basis to argue that a bank's reporting conventions cause this seasonality, we use the raw data in our implementation to err on the side of conservatism. We assume that the quarterly patterns in the Exhibit 3 average bank represent real seasonal fluctuations in net interest rates earned. Under this assumption, these seasonal patterns will be effectively smoothed out in the estimation Equation (9). This procedure produces lower R^2s and higher standard errors than would be obtained with the deseasonalized data.

Treasury data come from the University of Houston fixed-income database.[15] All Treasury bill, note, and bond data corresponding to the quarter dates in the Call Reports are collected. From these term structures, smoothed forward rate curves and extended Vasicek parameters are estimated.

III. STATISTICAL ANALYSIS

The first step in valuing the credit card portfolios is to generate smoothed forward rate curves and to estimate the term structure evolution parameters given in Equation (6).

Smoothed Forward Rate Curves

A piecewise constant function is used to construct smoothed forward rate curves. We use these curves primarily for simplicity, although there is some evidence that piecewise constant forms work quite well in this regard (see Bliss [1996]). More sophisticated procedures are available (such as Adams and van Deventer [1994]), but the need to use more sophisticated procedures for term structure modeling is an unresolved topic. We leave application of more sophisticated smoothing procedures to the valuation of credit card loans to subsequent research.

Using the Treasury bill, note, and bond data, we select the piecewise constant forward rate curve that best matches the market prices of the Treasury securities. We obtain this by first considering a coupon bond as a portfolio of zero-coupon bonds, and then using the relation between a zero-coupon bond and the forward rate. We choose the forward rates that minimize the sum of squared errors in matching the model's price to the market price at each quarterly date. Schwartz's [1998] outlier procedure is used to remove inconsistent prices.

Extended Vasicek Parameters

We use a historic (as opposed to implicit) estimation procedure to determine the extended Vasicek parameters (a, σ) from Equation (6). We adopt the procedure in Heitmann and Trautmann [1995] who estimated these parameters for German bond data (later used by Henn [1997] for U.S. Treasury data).

It is well known that under the extended Vasicek model:

$$\mathrm{var}(\log(P(t+\Delta, T)/P(t,T)) - r(t)\Delta) = \sigma^2\left(e^{-a(T-t)} - 1\right)^2 \Delta / a^2 \tag{13}$$

EXHIBIT 4

Extended Vasicek Parameters

Parameter	Estimate
σ	0.0076 (0.0034)*
a	0.0154 (0.0091)*

Estimated over January 1973-March 1997 using monthly observations of Treasury rates.

**Significantly different from zero at the 95% confidence level.*

Equation (13) represents the variance of the continuously compounded return on the T-th maturity zero-coupon bond less the spot rate over the time period $[t, t + \Delta]$.

Using the time series of zero-coupon bond price observations obtained from the smoothed forward rate curve, we compute the sample variance of the left-hand side of Equation (13) over the entire observation period. We run a cross-sectional non-linear regression across the different maturities to estimate the parameters (a, σ):

$$v_i = \sigma^2\left(e^{-ai} - 1\right)^2 \Delta / a^2 + e_i \quad \text{for all } i \tag{14}$$

where the v_i are the sample variances on the left-hand side of Equation (13) for the i-th maturity bond, and the e_i are independent and identically distributed errors with zero means and constant variances.

The estimates for these parameters are given in Exhibit 4. The mean-reversion coefficient $a = 0.0154$, and the volatility $\sigma = 0.0076$. These estimates are similar to those obtained by Henn [1997]. As indicated by Equation (6), this implies that short-term forward rates are more volatile than long-term rates.

Credit Card Loan Evolution Estimation

A non-linear regression is used to estimate the credit card loan regression equation for the five banks, as the sample size is small.[16]

The regression coefficients provide the desired estimates of the parameters (γ^Δ, μ, α, c) as given in Equation (9). These estimates and their standard errors are recorded in Exhibit 5. We report γ^Δ rather than γ because γ^Δ is interpreted as the growth rate adjustment, per Δ time periods, to the long-run credit card balances.

The results show that the regression equation pro-

EXHIBIT 5
Regression Estimates

Bank	γ^{Δ} (std. error)	c (std. error)	∝ (std. error)	μ (std. error)	R^2	h#	ρ	N
Average	0.78985* (0.08707)	1.14203* (0.33344)	0.07057* (0.01674)	6.67119 (12.13210)	0.95648	-0.80246	N/A	47
Chase Manhattan	0.78315* (0.05460)	7.34617 (4.84417)	0.06543 (0.04435)	12.34350 (27.04749)	0.981784	-2.17925	0.34	47
Vermont	0.68785* (0.09966)	1.29992* (0.27230)	0.11609* (0.01311)	-1.47753 (8.90631)	0.97929	-1.28745	N/A	47
Bank of Delaware	0.36715* (0.12991)	1.17066* (0.13105)	0.05033* (0.00689)	0.15021 (5.01927)	0.86242	-0.41139	N/A	47
First USA	0.79157* (0.08587)	1.87507* (1.03895)	0.16079* (0.03579)	-15.65501 (23.93409)	0.97225	0.08320	N/A	42
MBNA	0.06133 (0.23183)	0.73353* (0.05730)	0.21767* (0.01456)	8.88519 (7.95006)	0.95352	-0.21085	N/A	20

The regression is $\text{Log}L(t\Delta) = \gamma^{\Delta}\text{Log}L((t-1)\Delta) + (1-\gamma^{\Delta})\text{Log}(cL_0) + \gamma^{\Delta}\mu\Delta + \mu(1-\gamma^{\Delta})t\Delta + a\Delta(1-\gamma^{\Delta})R(t\Delta)$.

N/A: Not adjusted.

h-test statistic for autocorrelation after correcting for non-zero autocorrelation. If an adjustment is necessary, the estimated correlation (ρ) is given in the next column.

**Significantly different from zero at the 95% confidence level.*

vides a good fit to the data, with a high R^2 for all six banks (five banks plus the average). Second, most of the parameters (γ^{Δ}, μ, c) are significantly different from zero. Only Chase Manhattan needed an autocorrelation adjustment. The quality of the fit is quite high, indicating the importance of the growth rates (γ^{Δ}, μ).

The interest rate sensitivity coefficient (α) appears to be insignificantly different from zero for all six banks. Recall that the sign of this parameter is indeterminant by theory. It is expected to be positive if the bank's credit card borrowers add to their unpaid balances as the average rate rises, and negative if the opposite is true. Because the borrower pool differs across banks, for geographic and credit card selection reasons, the sign of this parameter may differ across banks as well.

This bank-specific sensitivity of unpaid balances to interest rate movements appears to be reflected in the data. Indeed, Chase Manhattan, the Bank of Delaware, and MBNA have positive α, while Vermont and First USA have negative α. The average bank's α is positive, although insignificantly different from zero.

It is interesting to note that the growth rates of credit card loans in all the banks are positive. The Bank of Delaware had the lowest growth rate of 0.05033 per year, and MBNA had the highest growth rate of 0.21767. We saw these differences in Exhibit 2.

The speed of adjustment to steady-state credit card balances as reflected in γ^{Δ} is less than 1.0 for all banks. This is consistent with the initial restriction on this parameter. The higher this quantity, the faster the movement toward steady-state balances. As indicated, Chase Manhattan and First USA are adjusting more quickly to steady-state than are Vermont, the Bank of Delaware, or MBNA. The average bank's adjustment factor is quite high at 0.78985.

The estimate of the steady-state credit card loan portfolio (c) is greater than 1.0 for all banks except MBNA (at 0.73353). This implies that steady-state balances exceed current balances for all banks except MBNA. The average bank's credit card loan portfolio is almost at steady-state (c = 1.14203).

Rate Earned Evolution Estimation

To estimate the net interest rate earned regression in Equation (11), again, due to the small sample size, we use a non-linear regression.

EXHIBIT 6
Regression Estimates

Bank	π^{Δ} (std. error)	k (std. error)	β (std. error)	R^2	DW	ρ	N
Average	0.15248 (0.13381)	0.08620* (0.00990)	1.87595* (0.62251)	0.99	1.84401	N/A	47
Chase Manhattan	0.37361* (0.13550)	0.17119* (0.01881)	-0.88136 (1.18738)	0.99	1.96495	0.1140	47
Vermont	0.11987 (0.14595)	0.05576* (0.01350)	2.21338* (0.84778)	0.99	1.73422	N/A	47
Bank of Delaware	0.64150* (0.11391)	0.12919* (0.04911)	1.04544 (3.09932)	0.97	2.16240	N/A	47
First USA	0.03594 (0.15632)	0.07779* (0.02523)	-0.07609 (1.64204)	0.99	1.95446	N/A	42
MBNA	0.30698 (0.20943)	0.16599* (0.02360)	-2.40359 (2.12524)	0.99	2.26643	N/A	20

The regression is $n(t\Delta) = \pi^{\Delta}n((t-1)\Delta) + (1-\pi^{\Delta})k + \beta\Delta(1-\pi^{\Delta})R(t\Delta)$ *where* $n(t) = i(t) - l(t)$.

N/A: Not adjusted.

DW: Durbin-Watson test statistics after correcting for non-zero autocorrelation. If an adjustment is necessary, the estimated correlation (ρ) is given in the next column.

**Significantly different from zero at the 95% confidence level.*

The results of fitting the net interest rate earned regression are shown in Exhibit 6. We can see that the regression provides a good fit to the data, with a high R^2 for all six banks. Only Chase Manhattan bank required an autocorrelation adjustment.

As Ausubel [1991] notes, the net interest rate earned equations do not vary significantly through time. For all banks, at least one of the two parameters (π^{Δ}, β) is insignificantly different from zero. When parameters are significant, however, the point estimates for the interest rate sensitivity parameters (π^{Δ}, β) are all positive.

Exhibit 6 shows that $\pi^{\Delta} < 1$ for all banks as required by the initial parameter restrictions. In addition, except for the Bank of Delaware and Chase, it is insignificant from zero. This suggests that the net interest earned may be near its steady-state level.

The differing sign of β across banks and its insignificance from zero is consistent with differing borrower pools across banks. For Chase Manhattan, First USA, and MBNA, the sign is negative; i.e., as average rates rise, the net interest earned on credit card balances appears to decline.[17] As average rates rise, the reverse happens for Vermont and the Bank of Delaware. The average bank's β is significant and positive (1.87595), indicating that as rates rise, the net spreads earned increase.

The steady-state net interest rate earned (k) is the lowest at 0.05576 for Vermont National Bank; 0.08620 for the average bank; and the highest at 0.17119 for Chase Manhattan. The fact that this steady-state spread is positive indicates that issuing credit card portfolios should at the limit earn monopoly rents. This observation is consistent with the segmented markets hypothesis for the banking industry discussed in Jarrow and van Deventer [1998].

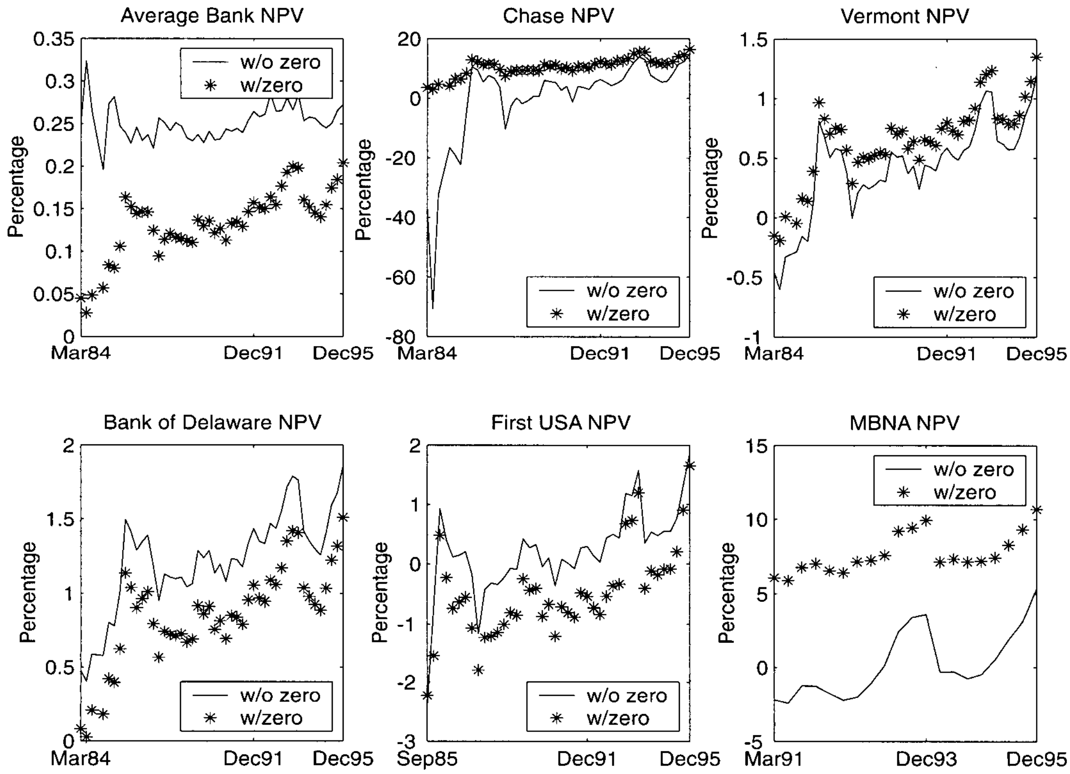
Credit Card Loan Valuation Estimates

Given the credit card balance and rates earned evolution parameters as estimated, we determine the net present value of the credit card loan portfolios. In this computation, the parameter values given in Exhibit 4 for the term structure of interest rates and in Exhibits 5 and 6 for the various accounts are fixed for the entire time period. The forward rate curve, however, varies across time.

Two sets of estimates are provided. The first set uses the point estimates for the interest rate sensitivity parameters (α, β) as given in Exhibits 5 and 6. The second set is for the interest rate sensitivity parameters set identically equal to zero when the estimates are statistically insignificant from zero.

EXHIBIT 7

Credit Card Loan Portfolio Net Present Values (as a percent of portfolio value)



W/o zero – Percentage NPV estimates using the point estimates of (α, β) *as given in Exhibits 5 and 6.*
W zero – Percentage NPV estimates using $(\alpha, \beta) = (0, 0)$.

The percentage credit card loan premiums are plotted in Exhibit 7 for the six banks. Summary statistics of the percentage credit card portfolio premiums are shown in Exhibit 8. Also included in Exhibit 8 is the average spread $[n(t) - i(t)]$ on the bank's credit card portfolio over the observation period.

Credit card portfolios appear to be becoming more profitable over the sample period for all banks except perhaps First USA. The increase in the banks' percentage NPV appears to be correlated with the growth in the credit balances as reflected in Exhibit 2. This correlation occurs because the spot rate is declining over this same period (*see Exhibit 3*), implying that the present value of a relatively constant spread earned is increasing.

Also, notice that the percentage NPV differs significantly depending upon whether the interest rate sensitivity parameters (α, β) are included or not (*see also Exhibit 7*). This differential value reflects the correlation of the balance growth (α) and net rates earned (β) with spot rate movements. Depending on the signs of the two parameters, this correlation can either diminish value (as with Chase Manhattan, Vermont, and MBNA) or augment value (as with the average bank, the Bank of Delaware, and First USA).

From Exhibit 8 one can see that for all banks the average percentage NPV of its credit card portfolio exceeds the average spread (the next-to-the last column). First, consider the average bank. The mean NPV is 0.2500 for the original point estimates (α, β) and 0.1334 if α is equal to zero. These NPV estimates reflect the growth rate of the credit card balances ($\mu = 0.07057$), the risk- adjusted discount rates, and the long-run credit card balances ($c = 1.14203$). They also reflect the slow movement ($\pi = 0.15248$) toward a steady-state rate earned of ($k = 0.0862$). We would thus expect the NPV of the average bank's credit card portfolio to exceed the average spread earned per period of 0.0544 in Exhibit 8.

In addition, both these estimates are similar in magnitude to those provided in Ausubel [1991] on resales of various credit card accounts between banks over the period

Exhibit 8
Summary Statistics of Credit Card NPVs

Bank		Mean	Std. Dev.	Max.	Min.	*Avg[n(t) – r(t)]*	N
Average	(α,β)	0.2500	0.0207	0.3234	0.1956	0.0544	47
	$(\alpha = 0)$	0.1334	0.0395	0.2040	0.0278		47
Chase	(α,β)	0.2658	15.2679	15.1736	-70.6844	0.0946	47
	$(\alpha = 0, \beta = 0)$	10.5942	3.0201	16.4368	3.0381		47
Vermont	(α,β)	0.4332	0.3898	1.2046	-0.6036	0.0296	47
	$(\alpha = 0)$	0.6538	0.3413	1.3505	-0.1936		47
Delaware	(α,β)	1.2375	0.3240	1.8585	0.4043	0.0873	47
	$(\alpha = 0, \beta = 0)$	0.8609	0.3303	1.5099	0.0262		47
FirstUSA	(α,β)	0.2134	0.7115	1.8710	-2.3734	0.0177	42
	$(\alpha = 0, \beta = 0)$	-0.4800	0.7625	1.6526	-2.2251		42
MBNA	(α,β)	0.2424	2.2999	5.3026	-2.4134	0.0979	20
	$(\alpha = 0, \beta = 0)$	7.6882	1.3352	10.6727	5.8805		20

1984-1990. The average resale premiums reported by Ausubel equal 20%, and all premiums reported are between 3% and 27%. This comparison indicates that the NPVs based on the point estimates of (α, β) provide the better estimates.

Delaware appears to be the most profitable bank, with a mean NPV of 1.2375 using the NPVs based on the point estimates of (α, β). Given the high standard error in the point estimates for Delaware's parameters and the small sample size, this is probably an overestimate of the premium earned. This is confirmed by Delaware's mean NPV of 0.8609 when both (α, β) are set equal to zero.

No banks appear to be losing money on their credit card loan portfolios, except perhaps First USA. But this is only when the point estimates for (α, β) are set equal to zero. In this case, First USA has a mean NPV of –0.48. This is due to the saw-toothed pattern in its net interest earned as contrasted with the spot rate of interest (*see Exhibit 3*).

As the sample size is small for these estimates, and the Federal Reserve Call Report data may have reporting errors, the exact magnitudes of an individual bank's NPV should be taken with a grain of salt. Of the estimates provided, we are most confident in the *average bank*'s NPV, because any reporting errors are likely to cancel out in the averaging procedure.

As a specification check, we also estimate the NPV assuming that all banks use the year-to-date convention for reporting losses and recoveries. In this case, the estimates remain highly significant, but the average bank's NPV from credit card lending is reduced somewhat to 21.50. Despite the imprecision of the individual bank estimates, our results provide a necessary comparison and help confirm the general validity of the model.

IV. CONCLUSION

We have developed the first tractable model for valuing credit card loan portfolios: an arbitrage-free valuation model that explicitly considers the evolution of credit card loan portfolio balances, rates earned, and realized losses. We estimate the model using quarterly Call Report data from 1984 through 1995.

Our results show that credit card loan premiums are roughly 25%. That is, a $1.00 credit card loan is worth $1.25 to the bank. As documented in Ausubel [1991], our estimates are consistent with the premiums obtained in recorded sales of credit card portfolios.

We hope the methods we have developed will prove useful for measuring and managing the risk exposure of credit card loans. It should be straightforward to extend this approach to other areas of consumer credit, such as auto loans, home equity loans, and personal lines of credit.

APPENDIX

Explicit Formulas for Deterministic Functions in Equation (12)

Define $\rho(s, t) = \sigma e^{-a(t-s)}$ and $b(s, t) = \sigma\left(1 - e^{-a(t-s)}\right)/a$

The formulas are:

$$\mu_1(t) \equiv \alpha \int_{\max[0,\, t-\Delta]}^{t} f(0, s)ds + \alpha \int_{\max[0,\, t-\Delta]}^{t} \frac{b(0, s)^2 ds}{2} - \int_0^t f(0, s)ds - \int_0^t \frac{b(s, t)^2 ds}{2}$$

$$\sigma_1^2(t) \equiv \int_0^{\max[0,\, t-\Delta]} \left[\alpha\frac{\left(e^{a\Delta} - 1\right)}{a}\rho(s, t) - b(s, t)\right]^2 ds + \int_{\max[0,\, t-\Delta]}^{t} [\alpha - 1]^2 b(s, t)^2 ds$$

$$\mu_2(t) \equiv \beta \int_{\max[0,\, t-\Delta]}^{t} f(0, s)ds + \beta \int_{\max[0,\, t-\Delta]}^{t} \frac{b(0, s)^2 ds}{2}$$

$$\sigma_2^2(t) \equiv \beta^2\frac{\left(e^{a\Delta} - 1\right)^2}{a^2} \int_0^{\max[0,\, t-\Delta]} \rho(s, t)^2 ds + \beta^2 \int_{\max[0,\, t-\Delta]}^{t} b(s, t)^2 ds$$

$$\sigma_{12}(t) \equiv \alpha\beta\frac{\left(e^{a\Delta} - 1\right)^2}{a^2} \int_0^{\max[0,\, t-\Delta]} \rho(s, t)^2 ds - \beta\frac{\left(e^{a\Delta} - 1\right)}{a} \int_0^{\max[0,\, t-\Delta]} \rho(s, t)b(s, t)dt + (\alpha - 1)\beta \int_{\max[0,\, t-\Delta]}^{t} b(s, t)^2 ds$$

$$\mu_3(t) \equiv f(0, t) + \frac{b(0, t)^2}{2}, \qquad \sigma_3^2(t) \equiv \int_0^t \rho(s, t)^2 ds$$

$$\sigma_{13}(t) \equiv \alpha\frac{\left[e^{a\Delta} - 1\right]}{a} \int_0^{\max[0,\, t-\Delta]} \rho(s, t)^2 ds - \int_0^{\max[0,\, t-\Delta]} \rho(s, t)b(s, t)ds + (\alpha - 1) \int_{\max[0,\, t-\Delta]}^{t} b(s, t)\rho(s, t)ds$$

ENDNOTES

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[1]See Durkin [2000] and Federal Reserve data series H.8. Formally, these numbers are based on "revolving consumer debt." Revolving consumer debt is primarily bank-issued credit card debt, but also includes private credit cards issued by department stores. Most other consumer debt is termed "nonrevolving" and includes items such as automobile and mobile home loans. Loans secured by real estate are not considered to be part of consumer credit.

[2]See *Newsweek,* August 27, 2002, p. 37.

[3]See the BIS publications, "Principles for the Management of Credit Risk" [2000], and "The New Basel Capital Accord, Third Consultives Paper" [2003] (www.bis.org). For background discussion, see Santos [2000].

[4]A finite (but long) trading horizon is a standard hypothesis necessary for the operation of the martingale measure that we use for valuation (see Jarrow and Madan [2000]).

[5]The factors must have continuous sample paths so that they can be represented by Brownian motion.

[6]Linear growth in logarithms implies exponential growth in levels.

[7]This can be seen by taking the limit as $t \to \infty$ in Equation (7), noting that $\gamma < 1$, setting $\mu \equiv 0$ and $r_t = r \equiv 0$. Then $\lim_{t\to\infty} L(t) = cL_0$.

[8]This discretization uses the assumption that $R(t) = R(t - \Delta) + u_t$ where u_t is an error term. This assumption is consistent with Equation (6) as explained in Janosi, Jarrow, and Zullo [1999]. Empirically, Janosi, Jarrow, and Zullo [1999] show that one cannot reject the hypothesis that $R(t)$ follows a random walk. In addition, mean-reversion under the martingale measure is supported by the evidence documenting the usefulness of the extended Vasicek model in the pricing of interest rate derivatives (see James and Webber [2000]).

[9]This is seen by noting that $\lim_{t\to\infty} n(t) = k$ when $r = 0$ and $\pi < 1$.

[10]For a proof, see Janosi, Jarrow, and Zullo [1999].

[11]Virtually all commercial banks in the U.S. fill out a quarterly Call Report where they report certain financial figures based on their operations.

[12]The time periods and name changes for First USA are: 1985:3-1987:3 MBANK USA; 1987:4-1989:3 LOMAS USA; and 1989:4-1995:4 First USA.

[13]Although reports are generally consistent, numbers from the Call Reports differ somewhat from commonly cited numbers like those in Asset-Backed Securities Weekly.

[14]A study of the data suggests there may be inconsistent reporting of losses and recoveries. According to the Call Report forms, banks should report these data items on a year-to-date basis. Of the five banks we examined, only First USA consis-

tently appears to follow this convention. Vermont National appears to follow the convention for part of the sample period, while Chase, Bank of Delaware, and MBNA clearly do not. These banks appear to record their information on losses and recoveries on a quarterly accrual basis, not a year-to-date basis.

[15]See www.uh.edu/~awarga/lb.html for more details. Unfortunately, Treasury data were missing for the month 1984: 12-30.

[16]The initial point for the non-linear regression is chosen through a grid search over the four-dimensional surface (γ^{Δ}, c, μ, α). This is to better insure that a global minimum is obtained in the regression procedure.

[17]This is consistent with the idea that these three banks aggressively grew their credit card portfolios in a declining interest rate environment.

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